Non-linear Processing to Remove Mullti-beam Sonar Artifacts

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ABSTRACT

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A simple non-linear technique for acoustic array beamforming, originally developed for enhanced seismic interpretation, is adapted and applied to high frequency multi-beam sonar. Elementary properties of the resultant algorithm are explored in a non-exhaustive manner using both theory and numerical simulation. Application to fisheries field data gathered by a Simrad-Mesotech SM 2000 multi-beam imaging sonar illustrates the technique's potential for the visual suppression of unwanted reverberation noise and the enhancement of apparent directivity over conventional linear beamforming.

RÉSUMÉ

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Une technique non linéaire simple de conformation de faisceau d'un réseau acoustique, originalement mise au point pour l'interprétation sismique améliorée, est adaptée et appliquée au sonar multi-faisceaux haute fréquence. Les propriétés élémentaires de l'algorithme résultant sont examinées de façon non exhaustive, à l'aide d'une simulation théorique et numérique. L'application à des données de zone de pêche, recueillies au moyen d'un sonar d'imagerie multi-faisceaux Simrad-Mesotech SM 2000, illustre le potentiel de la technique pour la suppression visuelle du bruit de réverbération non désiré et l'amélioration de la directivité apparente par rapport à la conformation de faisceau linéaire ordinaire.

PREFACE

This report is an expansion of technical information submissions treating the processing of multi-beam sonar data, circulated to Kongsberg Simrad-Mesotech Ltd. and the Dept. of Geodesy and Geomatics Engineering, University of New Brunswick, during the time period Oct. 1999 to March 2000. In the present report numerical modeling is extended to the "latitudinal" in addition to the "longitudinal" circular array response and a more comprehensive explanation of the algorithm's underlying mechanism is attempted. A previous inaccuracy in modeling the longitudinal response has been corrected. This problem did not extend to the processed field data examples, which are included in their original form.

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INTRODUCTION

Traditional beamforming algorithms applied to line or planar circular arrays use time or phase delay stacks of linear form to select signals arriving from a specified azimuth (Knight et al. 1981). For some time DFO has been analyzing the theoretical and real-world performance of circular array based multi-beam sonars with the objective of both qualitative and quantitative fisheries applications. A number of signal processing variants have been explored. A specific non-linear algorithm (Kanasewich et al. 1973), adapted and applied to sonar field data, appears to both enhance sonar angular resolution and strongly reject unwanted reverberation noise. A preliminary study of the properties and mechanism of this algorithm is undertaken.

The algorithm is developed and applied to field data collected by a prototype Kongsberg Simrad-Mesotech SM 2000 multi-beam sonar. This 200 kHz fully quantitative imaging sonar is characterized by an 80-element array distributed over an 180^0 circular arc of 9.37 cm radius. A detailed theoretical analysis of conventional beamforming and system quantification as applied to a commercial version SM 2000 incorporating identical elements over a 155^0 arc of slightly larger radius is treated by Cochrane et al. (unpublished data)[1]. The reader is referred to this more comprehensive document, which should be openly available shortly, for a fuller explanation of linear operational theory, elemental characteristics (identical to prototype), quantitative calibration methodologies, and envisioned fisheries applications.

THEORY

LINEAR PROCESSING

The linear signal stack commonly used for sonar reception beamforming is of the form:

$$Sum(\theta_b, t) = \sum_{n=n_1}^{n_2} v_n(t)\, W(\theta_b, \theta_n)\, e^{ik\Delta l_n} \qquad (1)$$

$v_n(t)$ are (array) elemental voltage time series specific to array element n. Following common high frequency sonar practice these elemental voltages have been synchronously demodulated to baseband and expressed in complex form. n_1 and n_2 define the summation aperture of the array which, at most, extends over all array elements visible from infinity in beamforming direction θ_b.

$W(\theta_b, \theta_n)$ is the array window or "shading" function (Clay and Medwin 1977). It is designed to effect a desired compromise between main lobe width and side lobe suppression.

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[1] The manuscript: *Cochrane, N.A., Y. Li , and G.D. Melvin. Quantification of a multi-beam sonar for fisheries assessment applications* has been submitted to the Journal of the Acoustical Society of America.

The products $k\Delta l_n$ represent the phase rotations required to stack all array elemental signals in-phase in the beamforming direction. k is the acoustic signal wave number, $2\pi f /c$ where f is the acoustic frequency and c the propagation velocity in the surrounding medium. The Δl_n represent propagation path length differences measured from infinity in the beamforming direction to each element n. Expression (1) embodies the "monochromatic" or long pulse approximation, namely, that for given t, $\left|v_n(t)\right|$ varies with n only as a function of the transducer's orientation to the incoming pressure wave (its directivity response). The spatial variability inherent in the finite duration nature of the pulse envelope is ignored. This implies that for the complex baseband demodulated voltages easy-to-implement phase delays, alone, as opposed to more difficult-to-engineer precision time delays involving interpolation plus a phase rotation (Knight et al. 1981), are sufficient to ensure accurate stacking of waveform replicas of sonar signals arriving in the beamforming direction.

For a sound source at infinity in beamforming direction θ_b producing incident pressure signals $p_n(t)$ at each array element n, expression (1) can rewritten:

$$Sum(\theta_b,t) = K_{v/p} \sum_{n=n_1}^{n_2} p_n(t)\, D(|\theta_b - \theta_n|)\; W(\theta_b,\theta_n)\, e^{ik\Delta l_n} \qquad (2)$$

$D(\Delta\theta)$ represents the elemental directivity function (Medwin and Clay 1997), and $K_{v/p}$ an elemental (with associated electronics) on-axis receive pressure-to-voltage conversion factor (*ibid.*) assumed identical for all elements. Assuming identical p at all elements except for phase shifts compensated by the exponential term, expressions (1) and (2) may be equated to yield a beamformer estimator for p:

$$p(\theta_b,t) = \frac{\sum_{n=n_1}^{n_2} v_n(t)\, W(\theta_b,\theta_n)\, e^{ik\Delta l_n}}{K_{v/p} \sum_{n=n_1}^{n_2} D(|\theta_b - \theta_n|) W(\theta_b,\theta_n)} \qquad (3)$$

$p(\theta_b, t)$ is the beamformer output pressure (complex) time series while beamforming in direction θ_b. The denominator summation serves as a normalization factor for the elemental voltage stack. Normalization ensures identical p estimates for a source placed at any arbitrary beamforming direction θ_b and using any chosen functional form for $W(\theta_b, \theta_n)$.

NON-LINEAR PROCESSING

General Technique

To follow is an adaptation of a non-linear technique presented by Kanasewich et al. (1973) in the context of enhanced head wave phase velocity filtration of seismic time series received by a multi-channel array. The starting point was a linear velocity filter consisting of simple time domain shift and stack operations. The resultant velocity filter can be equivalently viewed as a directional beamformer designed to extract refraction survey head waves with a predefined emergence angle. The Kanasewich algorithm, denoted the "N th-root stack" is based on an earlier teleseismic array processing algorithm of Muirhead (1968). Both procedures are formulated for un-demodulated time series. The central idea is to time align the elemental time series, as in a normal time series stack, but rather than simply stacking their amplitudes, to alternatively stack the N^{th}-root of these amplitudes, preserving the original time series signs. A measure of linear-like amplitude restoration is subsequently introduced by raising the appropriately scaled, stacked N^{th}-root time series to the N^{th} power, again preserving sign.

Linearity as applied to acoustic array processing implies that two or more discrete time signals simultaneously arriving at an array yield an output equal to the sum of the output array responses for each of the signals arriving separately. A fundamental property of the Kanasewich - Muirhead procedure is that if a) no noise is present, b) all signal energy arrives from the beamforming direction, and c) all array elements are of equal sensitivity; then both linear and non-linear stacks yield equivalent results. In the more general case, the non-linear algorithm yields quite different results but with unique properties, useful in select applications.

Kanasewich et al. (1973) demonstrate that the non-linear technique strongly suppresses additive random noise confined either to a single channel or occurring incoherently across the array. Nevertheless, Brown and Rowlands (1959) have shown that, at very poor signal-to- (Gaussian additive) noise ratios, the information content extracted from an array cannot be enhanced by any type of processing over that obtained by simple signal addition (linear stacking). Kanasewich et al.'s demonstrative application to the velocity filtration of refraction seismograms suggests an enhanced ability to separate phases of contrasting velocity (i.e. apparent emergence angle) much like that achieved using conventional linear processing but applied to a physically longer array than was actually utilized. This hints at a non-linear property somewhat akin to array super-directivity (Pritchard 1953). However, the authors make no attempt to extend the analysis in this context. Enhanced directivity could potentially be useful in the analogous sonar context where this property might be beneficially traded-off against signal-to-additive noise or other less critical signal properties. This conjecture is explored as applied to the prototype SM 2000 multi-beam sonar. Properties of the non-linear algorithm are examined by numerical simulation followed by application to real field data.

Algorithm Adaptation

The initial step is to define a variant of the non-linear algorithm compatible with the baseband demodulated elemental signals logged by the SM 2000.

Consider the linear processing case embodied in expression (1). Define a boxcar summation window so that $W(\theta_b, \theta_n) = 1$ for an appropriately assigned summation range defined by n_1 and n_2. Let the necessary stacking phase shifts be combined with v_n to yield phase shifted complex time series, $x_n(t)$:

$$Sum(\theta_b, t) = \sum_{n=n_1}^{n_2} x_n(t) \tag{4}$$

In the N^{th}-root stack one preserves the instantaneous phase of $x_n(t)$ while dynamically compressing the time series by raising its amplitude to the $1/N^{th}$ power:

$$Sum_{Mod}(\theta_b, t) = \sum_{n=n_1}^{n_2} \left[\frac{x_n(t)}{|x_n(t)|} \right] * |x_n(t)|^{1/N} \tag{5}$$

$$Sum_{Mod}(\theta_b, t) = \sum_{n=n_1}^{n_2} x_n(t) * |x_n(t)|^{(1/N - 1)} \tag{6}$$

To produce the resultant output signal time series, $p(\theta_b, t)$, the dynamic compression is removed by raising the suitably scaled sum to the N^{th} power. This restores linearity for signals arriving precisely in the beamforming direction. In analogy with (3), the result of a suitably normalized restoration will be:

$$p(\theta_b, t) = \left[\frac{|Sum_{Mod}(\theta_b, t)|}{\sum_{n=n_1}^{n_2} D^{1/N}(|\theta_b - \theta_n|)} \right]^N \tag{7}$$

The action of the N^{th}-root stack, as defined above for a baseband demodulated data stream, differs from that as described by Kanasewich et al. for an un-demodulated time series. The latter investigators' procedure, at least for large N, converts the original time series into a new time series crudely approximating a near constant amplitude square wave. Now, consider the effect of the N^{th}-root operation applied to the demodulated signal, followed by conversion back to the original carrier frequency domain. The overall effect is to convert the original carrier signal, a sinusoid rapidly varying in

amplitude and phase, to a signal varying similarly in phase but, in general, only slowly in amplitude. The result is very similar to passing the output of Kanasewich et al.'s N^{th} -root operation applied to the original carrier frequency signal through a low pass filter constructed to remove the harmonics inherent in the square wave-like wave form. This operation would leave only the carrier fundamental with sufficient bandwidth to accommodate its modulation envelope.

THEORETICAL SIMULATIONS

RESULTS

Numerical simulations of the SM 2000 array's receive response afford insight into the algorithm action. Expressions for computation of both the equatorial and meridional response of the SM 2000 are contained in (Cochrane et al. unpublished data). Figs. 1 – 3 show beamformer amplitude responses for an isolated source moved in the array's equatorial plane through a fixed acoustic beam formed at $\theta_b = 90^0$, the responses computed using 3 contrasting types of processing. The 90^0 beam direction is defined by the vector pointing outward from the array center of curvature and passing through the physical array center (the center of array element 1 sits at 0^0 and the center of element 80 at 180^0). Fig. 1 shows the receive response for conventional linear processing using the full 180^0 array aperture shaded by a hamming "low side lobe" window (Cochrane et al. unpublished data) and incorporating individual elemental directivities. Fig. 2 shows the corresponding response when the hamming window is replaced by a boxcar window extending over the full array aperture while still retaining the elemental directivities. Moderate narrowing of the main lobe response is observed at the expense of significantly higher side lobes.

Fig. 3 shows the receive response of an 8^{th} -root stack, incorporating both the boxcar window and elemental directivities. A modest narrowing of the main lobe at the – 3 dB points is observed compared to the corresponding results from linear processing with similar window. For the non-linear processing the main lobe response skirts further from the beam axis are comparatively steeper. Side lobes are suppressed to the point of practical nonexistence.

The latitudinal receive response of the SM 2000 transducer is also interesting. Figs. 4 and 5 show the latitudinal response for the 90^0 beam using 180^0 "low side lobe" hamming and 180^0 boxcar windows respectively. Fig. 6 shows the corresponding non-linear algorithm latitudinal response for $N = 8$. The effect of the algorithm is to "square-up" the response main lobe, again with the practical elimination of all side lobes.

DISCUSSION

The properties of non-linear signal processors are easiest to study when they are constrained so as to approximate the behaviour of linear systems. The interpretation of the action of the N^{th}-root stack below is far from complete, at points quite speculative,

but the limited perspectives presented may open avenues for more thorough and rigorous investigation.

One observation using original carrier frequency, as opposed to baseband demodulated, time series is that taking a small fractional power (high root) of time series amplitudes while preserving sign is quite similar to the action of an analog clipping amplifier where the clipping action is somewhat "soft". The most accessible preserved information is that defined by the zero point crossings (i.e. phase information). However, since the clipping is soft, amplitude information is retained that is potentially recoverable by subsequent specialized waveform manipulation. As previously noted, the result of the N^{th}-root stack performed on the baseband signal is roughly equivalent to harmonic filtering of the clipping amplifier output. Also noted was the fact that for isolated signals arriving in the precise beamforming direction the results of linear and non-linear processing are identical – when properly normalized.

For the SM 2000 semi-circular array linearly beamforming at 90^0, utilizing all visible array elements, and a boxcar summation window; array element directional responses introduce an implicit shading function which acts to broaden the main lobe while lowering side lobe amplitudes compared to the case of omnidirectional array elements. This is illustrated in the receive response simulation in Fig. 7 where the boxcar window continues to be used but the original elements have been replaced by omnidirectional entities (the elemental response is considered zero when physically hidden to the source by array curvature). Side lobe suppression is now extremely poor – even poorer than for the equivalent-projected-length line array. The reason is that the semi-circular arc of transducers when projected onto a line normal to the beamforming direction results in non-equally spaced elements, the elements being more closely clustered near the line array ends. The result will be roughly equivalent to an equal length, close equi-spaced array with higher weightings given to the outer elements. The effect is to narrow the central lobe at the expense of very poor high side lobe suppression. Our contention is that the action of the non-linear algorithm precompression operation applied to the SM 2000 array, for larger values of N, is to equalize the elemental signals sufficiently as to effectively remove the implicit shading arising from the elemental directional responses.

The stacking operation following precompression as in Eq. (6), and without amplitude restoration, should yield a directional response form rather similar to the linear response of Fig. 7. This follows because approximately equi-amplitude sinusoids are being stacked in the non-linear case compared to precisely equi-amplitude sinusoids in the linear case. However, in the non-linear processing this core response form is subsequently raised to the N^{th} power resulting in the side lobe amplitudes dropping relative to the main lobe by a factor of N **in decibels**. Verification this conjecture is afforded in Fig. 8 where the full algorithm non-linear response curve of Fig. 3 is replotted at $1/N = 1/8$ th the original vertical scale. The graphical side lobe patterns of Fig. 8 and Fig. 7 physically superimpose closely.

The impression might be conveyed that the N^{th}-root stack in effect is essentially the same as taking the output time series from standard linear processing and raising it to the N^{th}

power. That this is not the case is illustrated by the comparative examples in Table 1. In the table, beams are formed using the 8^{th}-root stack and also by a normal linear stack with the latter's output raised to the 8^{th}-power. Two non-interacting sound sources occupy the beams. Source # 1 has a level sufficient to yield a beamformer output of unity for both 8^{th}-root and 8^{th}-power beamforming. Source # 2 has a source level 20 dB higher than source # 1. Table 1 lists approximate beamformer outputs expected with the sources individually placed alternately on-axis and at the –20 dB off-axis response angle appropriate to an omnidirectional element array linearly processed.

One observes that while both types of processing yield similar increases in "numerical" directivity (i.e. taking output magnitudes only, neglecting dimensionality), the 8^{th} power process achieves this only by way of greatly increasing the disparity between the on-axis responses for the two sources. Applied to an echogram, the 8^{th} power process serves only to increase the image's photographic contrast – not a very effective image enhancement procedure since weaker targets tend to be lost. In contrast, the 8^{th}-root process, by virtue of the additional precompression stage, achieves the equivalent increase in numerical directivity without increasing the disparity in on-axis levels. Note that in Table 1, the 8^{th} -root process preserves the factor of 10 in beamformer response between the two sources regardless of whether both sources are on axis or at the (linear) –20 dB angular point. This is a direct consequence of the N^{th}-root stack's inherent dimensional correctness. The action of the 8^{th}-root procedure more closely resembles true (linear) array super-directivity - although it is difficult to compare non-linear and linear processes. The corresponding echogram application is also much more effective since resolution is increased but the photographic contrast remains largely unaltered. Inclusion of the elemental directivities would modify these results to some degree but the principal characteristics should remain.

Any useful qualitative – as opposed to merely qualitative or "visual" – real-world application of the non-linear algorithm would depend upon the extent to which the linearity or near-linearity for isolated main lobe signals is preserved in the presence of strong signal energy arriving simultaneously from other directions. While these critical but complex interactions could either be modeled numerically or studied experimentally using controlled multiple source geometries, this has yet to be accomplished. Nevertheless, some indication of the potential of the non-linear algorithm may be gleaned by comparing the various beamforming methods using existing SM 2000 field data.

APPLICATION TO FIELD DATA

RESULTS

Since the SM 2000 is capable of logging elemental voltages in baseband form, comparative beamforming algorithms are readily studied in post-processing. Figs. 9 - 16 compare linear and non-linear processed field data. Field data was gathered from C.C.G.S. TELEOST on Browns Bank in the presence of aggregated herring. The sonar was towed at about 7 m depth by a long-starboard-reach boom. The sonar's equatorial plane was normal to the direction of tow. A selected receive data block corresponding to a single transmission is post-processed into 256 equi-spaced beam time series and polar

plotted as a 180^0 port-starboard vertical section echogram[1] for each type of processing, the 90^0 direction pointing straight down (nadir direction)[2].
Receive beamforming utilizes summation over all transducer elements visible from infinity in the beamforming with no window function shading applied in processing. Neither transmit response correction nor summation normalization is included in the processed sections. Receive beamwidths, between the –3 dB points, with linear processing, range from 2^0 for the near-nadir beam to about double this value for the horizontal beams. It will be noted that the horizontal beams are limited to one half the angular summation aperture of the nadir beam. On transmit, all array elements are fired simultaneously resulting in a broad equatorial plane pattern with significant drop-off only on approaching 90^0 from the nadir. The meridional transmit response is quite similar in form to the linear receive response of Fig. 4 (theoretical meridional transmit and receive response differ slightly, especially in their side lobes, due to the inclusion of stacking phase delays and a window function in the latter).

Fig. 9 shows the result of conventional linear processing for the chosen section. Figs. 10 - 15 show application of the N^{th}-root stack to the same section. Fig. 16 shows the result of an alternative "noise limiter" type beamformer technique described in the APPENDIX. A 40 log R receive TVG response is used. The maximum profiling range of 75 m corresponds to the outer edge of the printed sections. In all figures, reproduction contrasts have adjusted for best detail. The roughly horizontal layer at a range of 51 m is the bottom reflection. On the linear plot of Fig. 9 a broad and strong circular ring of artifacts is observed with onset time corresponding to the initial arrival of scattered energy from the bottom. Two weaker artifact rings are observed at ranges of 7 and 15 m. The latter two rings are believed due to reflected energy from the water surface and from the ship's wake respectively, the wake lying out of the sonar's equatorial plane. Numerous herring echoes appear, these being most highly concentrated between depths of 15 and 25 m from the top of the section..

DISCUSSION

Application of the N^{th}-root stack with N in the range of 4 to 8 removes virtually all visible bottom reflected noise (i.e. side lobe and probably cross-coupled preamp feed-through) which is seen to strongly contaminate the linearly processed data (Fig. 9) for synthesized beams sampling the water column at ranges greater than the transducer to bottom separation (51 m). However, for higher values of N, the non-linear methodology also appears to strongly suppresses real coherent signals embedded in this noise unless their amplitudes are sufficiently high (at individual elemental outputs) to be comparable to the noise amplitudes. This is most clearly seen in Fig. 15 ($N = 8$) where there appears to be a definite drop in legitimate water column signals at ranges greater than 51 m where strong bottom reverberation is expected on the relevant elemental signals.

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[1] The processed vertical section is ping #1764 (starting at #1) of file 97i20001.raw (CD SM975206) of TELEOST Cruise # 97-052.
[2] Since an odd number of beams are synthesized spanning $0 – 180^0$ no beam is synthesized at precisely 90^0.

The useful characteristic of narrow beam widths with virtually no side lobes, theoretically predicted for the N^{th}-root stack under restrictive conditions, may well be approximated under more general and realistic operational conditions. Such seems evidenced by the much reduced angular spread of discrete fish target echoes with the non-linear processing, as well as the much reduced "subbottom" reverberation from bottom side lobe received energy, and the rejection of mid-water artifacts from energy arriving out of the main lobe. As illustrative of the latter effect, note, in Fig. 9, the strong near-surface reverberation in the vicinity of the 180^0 beam (LHS of echogram) at 15 m range in the linearly processed section, arising from the off-equatorial-plane reflections from the ship's props or wake. In fact, detailed inspection of Fig. 9 reveals a trail of varying amplitude residual artifacts extending over a complete 180^0 arc. Some artifacts may be originating through beam side lobes and some from residual cross coupling effects in the receiver preamps (an analog design limitation). For larger values of N, the N^{th}-root stack algorithm removes these artifacts almost completely (Fig. 15). The "noise limiter" algorithm copes less well in this regard (Fig. 16). Similar observations can be made for the second weak arc of artifacts at about 7 m range due to water surface reflections. For this experimental data set, the N^{th}-root stack with an $N \approx 4$ seems a reasonable compromise between significant removal of non-main lobe originating artifacts and minimal simultaneous suppression of legitimate main lobe targets in the presence of the high off-beam-axis reverberation.

The non-linear N^{th}-root stack appears to be an effective technique for removing spurious artifacts from SM 2000 multi-beam sonar data, and consequently improving the visual appearance of echogram sections - especially when the signal-to-noise ratio (SNR) is high. However, it is doubtful if the non-linear algorithm reveals target signals embedded in noise (low SNR condition) that would not be evident under standard linear processing. Under high SNR conditions, and at least for single targets, an enhanced main lobe directivity over the linear case appears to be generated by the N^{th}-root stack processing (compare single target simulations in Figs. 1 & 2). In the general case of simultaneous signals from multiple targets, considerable beamformer waveform distortion might be expected from the resultant non-linear cross modulations. How severe these are and their effect on the linearity of signals arriving on-axis or and close to on-axis are critical questions yet to be resolved.

CONCLUSIONS

The N^{th}-root stack as applied to multi-beam sonar is seen to be a promising non-linear technique for achieving narrow beamformed main lobes and very high side lobe signal rejection under, at least, some restrictive conditions. Quantitative trade-offs in beamformer properties when applying the technique under more general conditions remain to be explored. Preliminary applications to field data show the N^{th}-root stack to have useful properties, at least, when utilized as a qualitative visualization tool. Variations of the technique should be applicable to a variety of sonar array applications involving water column and bathymetric imaging.

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Table 1. Approximate beamformer output amplitudes for two individual acoustic sources placed, first, on the 90^0 beam axis and, secondly, off-axis at the angular position of the –20 dB point of the omnidirectional element, linear receive beamformer response curve. Two beamformers are used, the 8^{th}-root stack with amplitude restoration, and a linear beamformer with output raised to the 8^{th} power. Omnidirectional array elements and boxcar window summations over all visible elements are assumed for both beamformers.

Source	On-Axis	-20 dB Point
# 1 0 dB	8^{th} root 1	10^{-8}
	8^{th} power 1	10^{-8}
# 2 + 20 dB	8^{th} root 10	10^{-7}
	8^{th} power 10^{8}	1

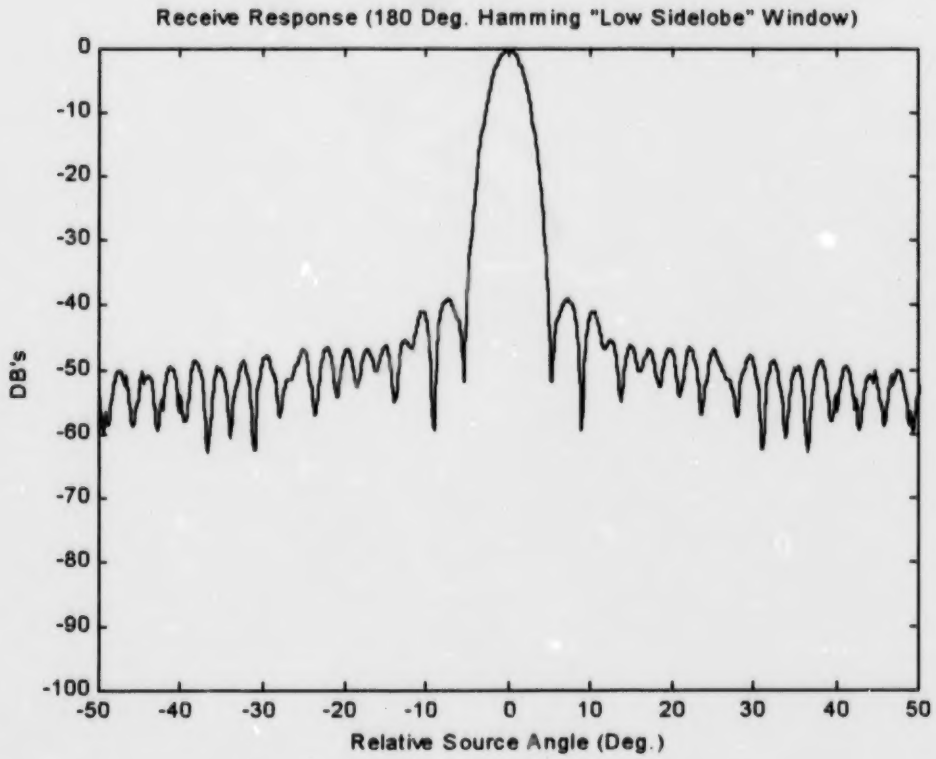

Figure 1. SM 2000 (prototype 9.37 cm radius 180° head) receive response in equatorial plane computed by normal linear stack. An SM 2000 "hamming low side lobe" window response extending over 180° of array summation aperture is used. Beamforming is at 90^0 (fixed) with moving source.

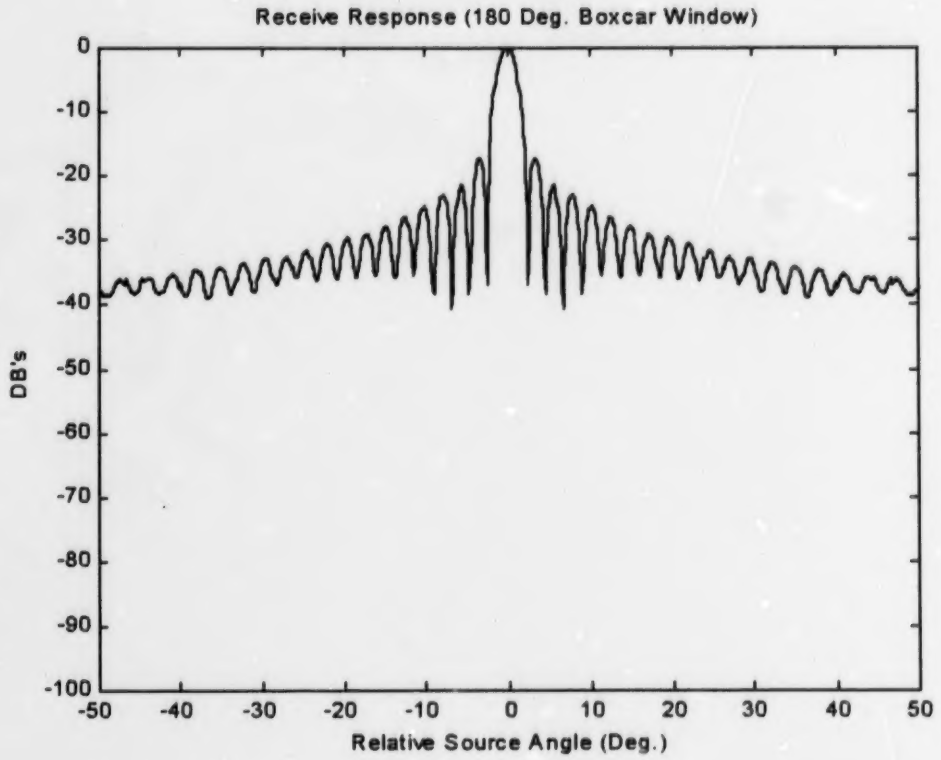

Figure 2. SM 2000 receive response in equatorial plane computed by linear stack using a boxcar window response extending over 180^{o} of array summation aperture. Beamforming is at 90^{0} (fixed) with moving source.

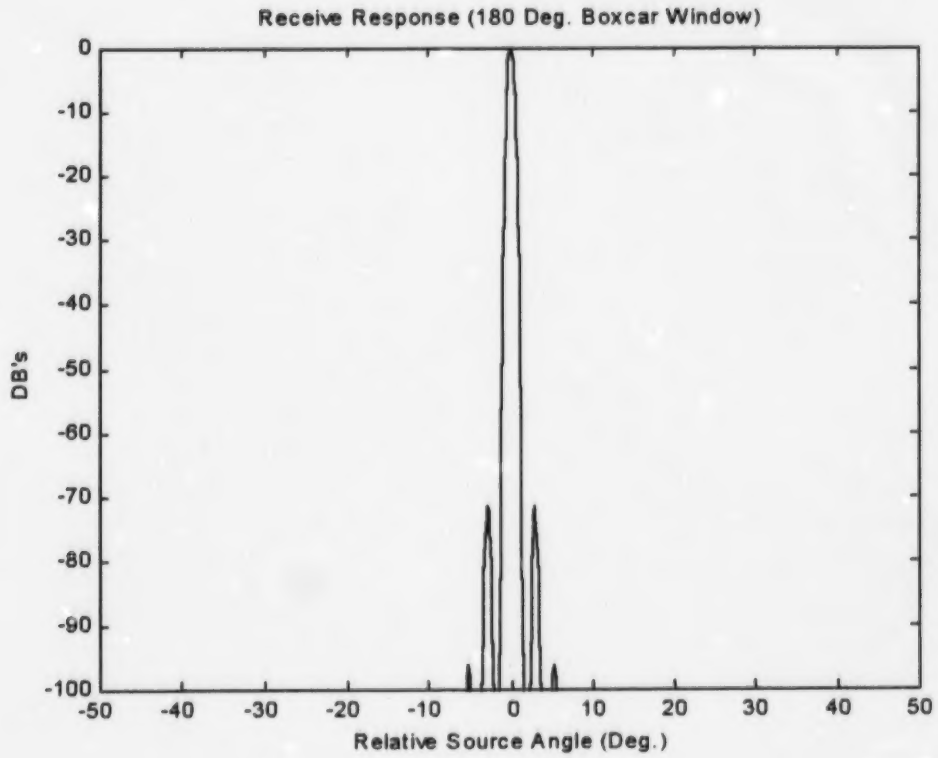

Figure 3. SM 2000 receive response in equatorial plane using 8^{th}-root stack and boxcar window extending over 180^{o} of array summation aperture. Beamforming is at 90^{0} (fixed) with moving source assuming infinite SNR.

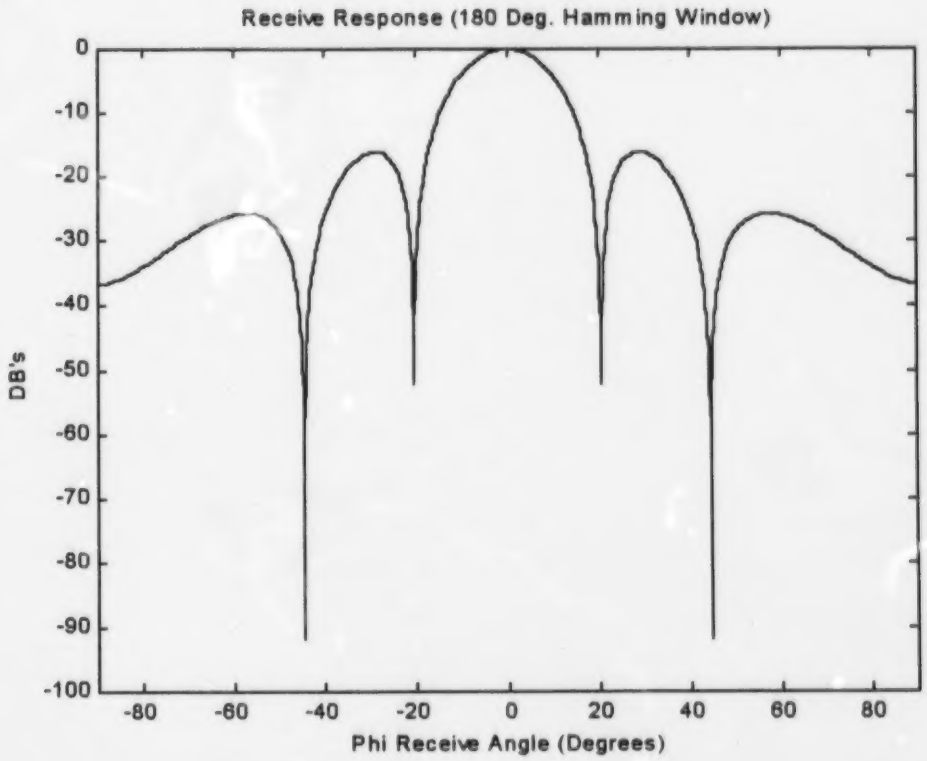

Figure 4. SM 2000 receive response (prototype 9.37 cm radius 180° head) along 90° meridian computed by normal linear stack using SM 2000 "hamming low side lobe" window response extending over 180° of (longitudinal) array summation aperture.

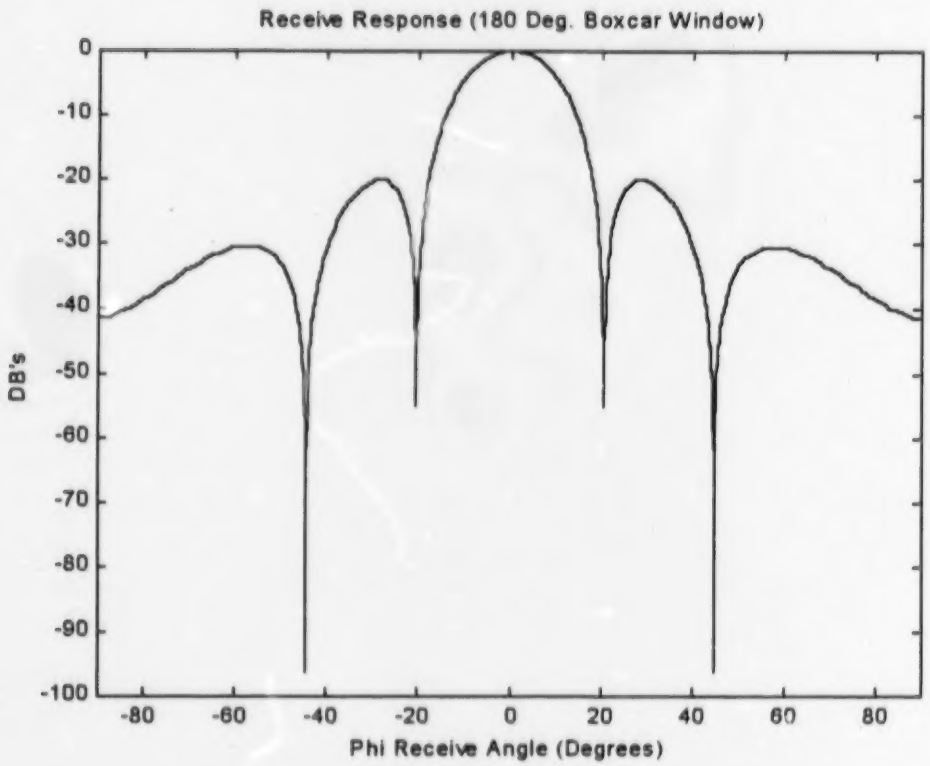

Figure 5. SM 2000 receive response (prototype 9.37 cm radius 180° head) along 90° meridian computed by normal linear stack using a boxcar window response extending over 180° of array summation aperture.

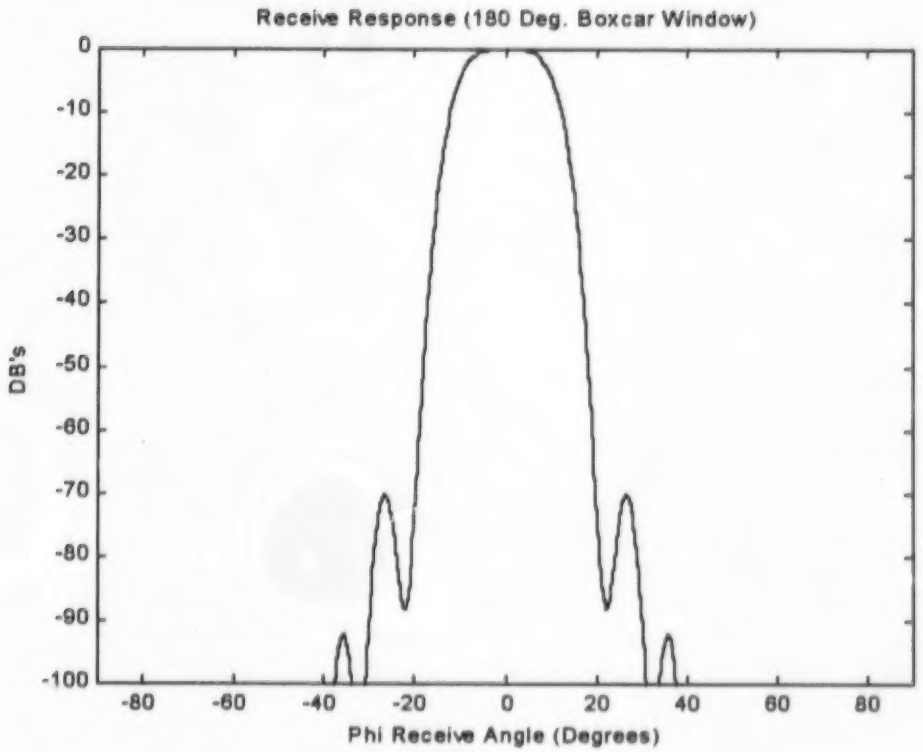

Figure 6. SM 2000 receive response along 90^0 meridian using 8^{th}-root stack and boxcar window extending over 180^o of array summation aperture. Infinite SNR is assumed.

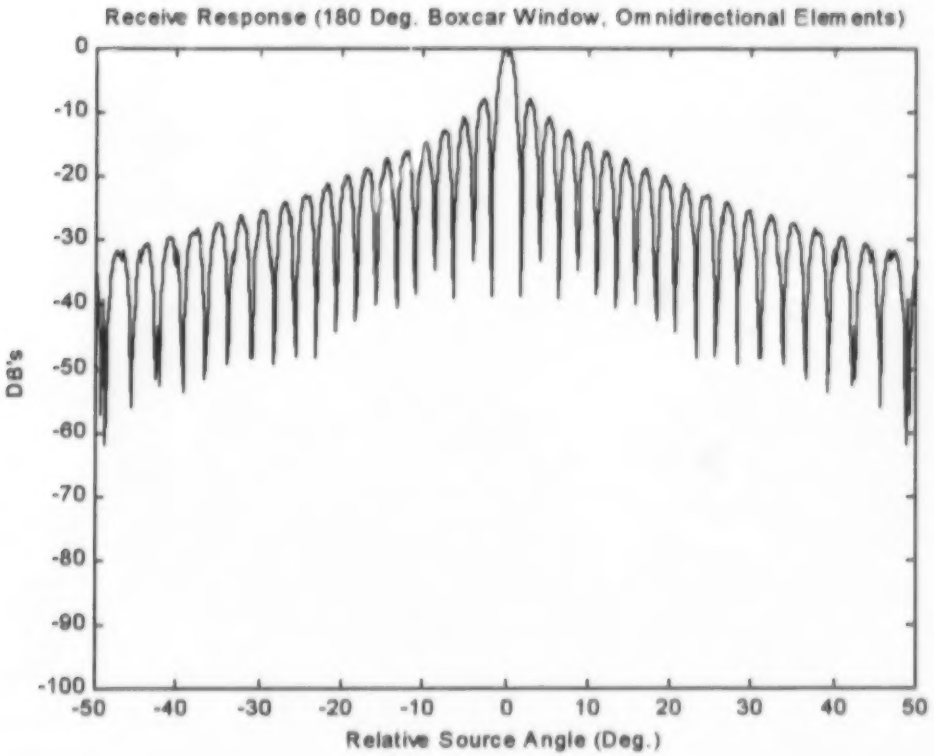

Figure 7. Hypothetical SM 2000 receive response in equatorial plane computed by linear stack using boxcar window response extending over 180° of array summation aperture. Omnidirectional as opposed to real directive array elements are assumed. Beamforming is at 90° (fixed) with moving target.

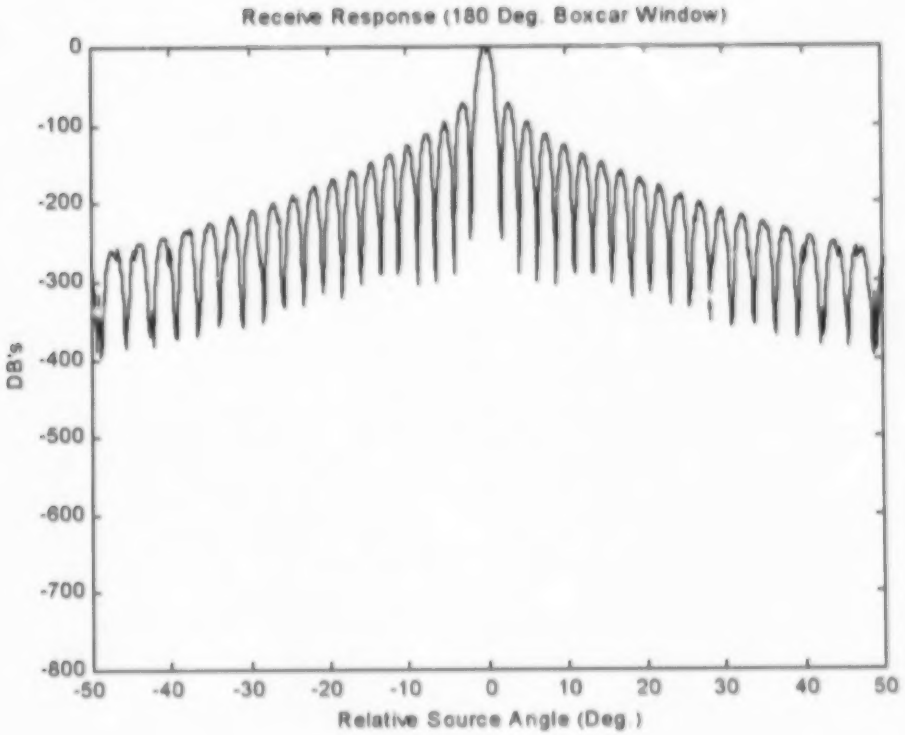

Figure 8. Non-linear receive response as ir Fig. 3 replotted at 8x original scale.

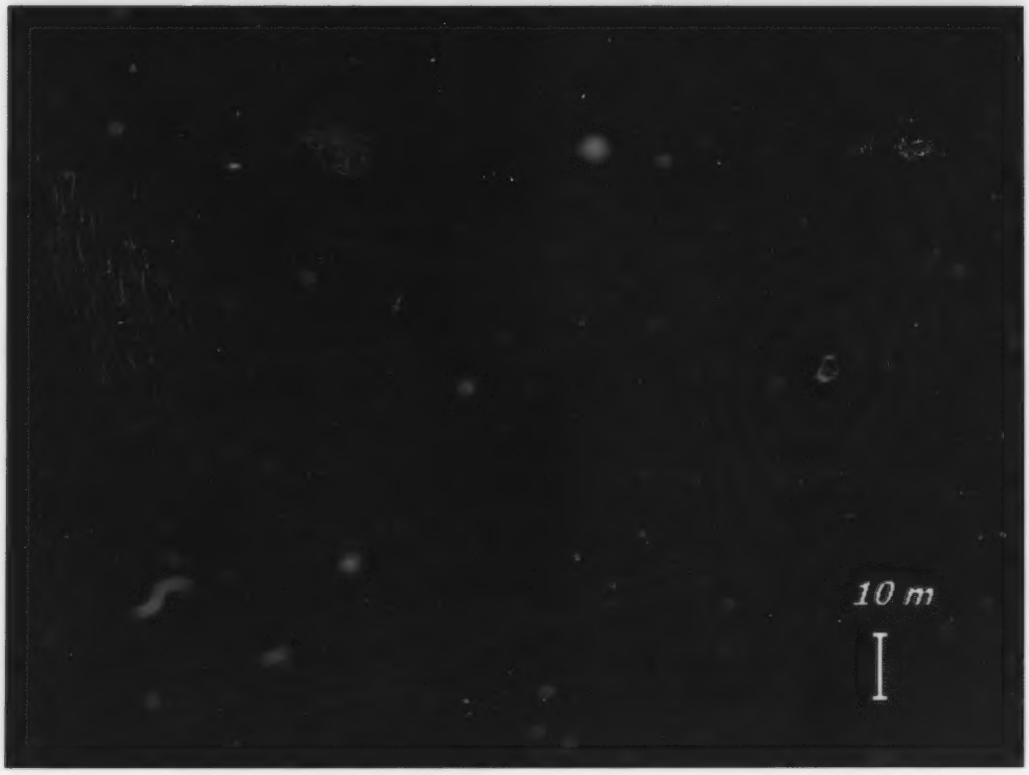

Figure 9. Vertical section echogram computed by normal linear processing. A hamming "low side lobe" 180° window is utilized, with full summation normalization for window function and individual elemental directivities.

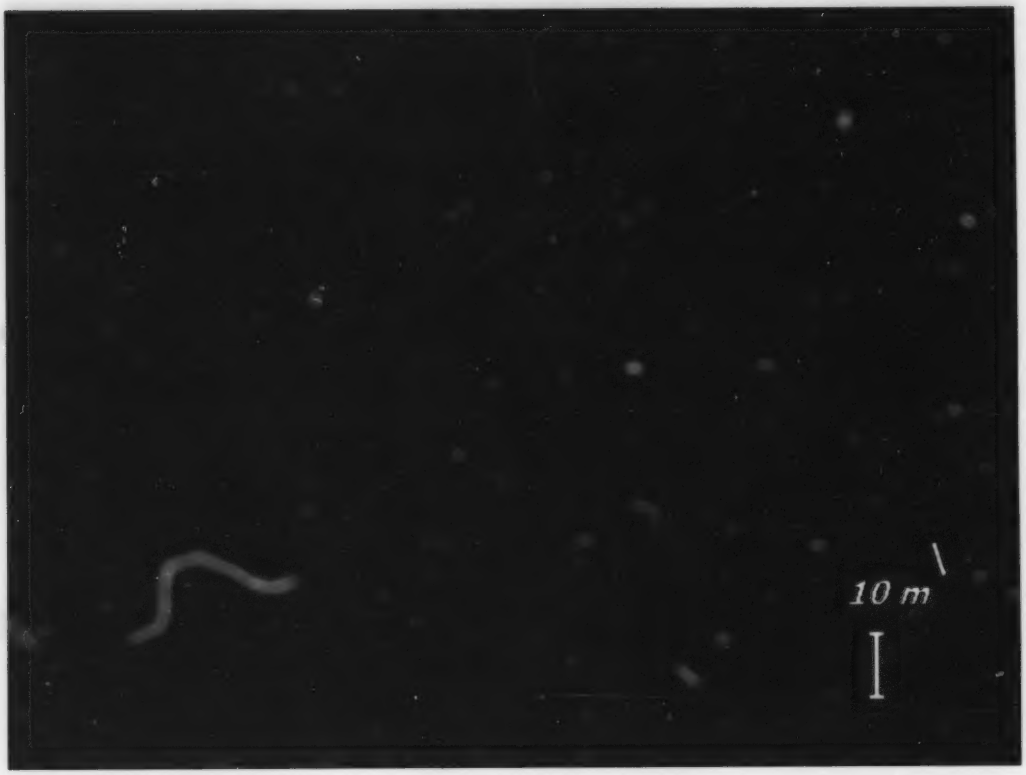

Figure 10. Square root stack. Boxcar 180° summation window utilized.

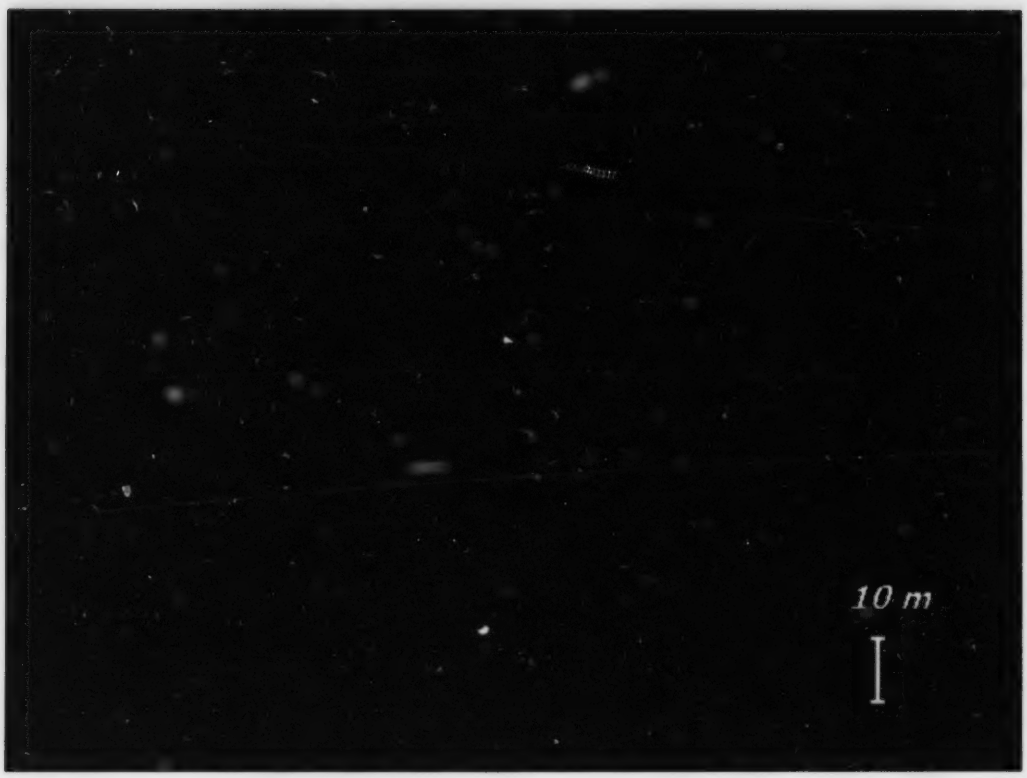

Figure 11. Cube root stack. Boxcar 180° summation window utilized.

Figure 12. 4^{th}-root stack. Boxcar 90^{o} summation window utilized.

Figure 13. 4^{th}-root stack. Boxcar 180^{o} summation window utilized.

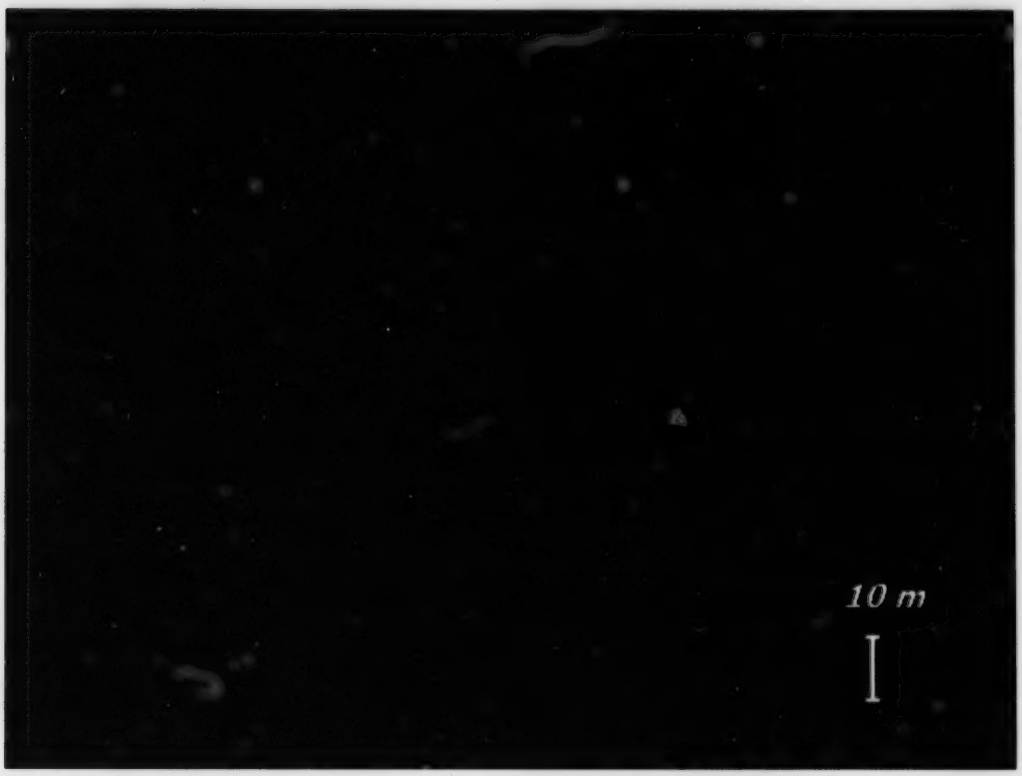

Figure 14. 6th-root stack. Boxcar 180° summation window utilized.

Figure 15. 8^{th}-root stack. Boxcar 180^0 summation window utilized.

Figure 16. Custom non-linear function. $a = 90.0$. Boxcar 180° summation window.

APPENDIX. ALTERNATIVE NON-LINEAR TECHNIQUE

An alternative non-linear noise reduction technique is to use a multiplicative signal amplitude dependant scaling factor applied to the complex elemental voltage time series, $x_n(t)$. This operation yields a rescaled time series, $y_n(t)$, of the form:

$$y_n(t) = \frac{a}{\left(|x_n(t)| + a\right)} x_n(t) \qquad \text{(A1)}$$

It will be noted that the original signal phase is retained in the product in analogy to the N^{th}–root stack. For an appropriately chosen value of a (about 90.0 for the data scalings typically employed with the SM 2000) the summation response is essentially linear for x amplitudes << a. Quantity a is chosen so as to include most typical fish echoes in the linear response region while reverberation noise with amplitudes approaching or exceeding a falls in the markedly non-linear response region. The modified $y_n(t)$ are beamformed by a normal linear processing stack. Again, in analogy to the N^{th}-root stack technique, an inverse response compensation can be applied to the average stack to remove, at least to a degree, the non-linearity for higher amplitude main lobe arriving signals. Compensation will be most exact when the signal-to-noise ratio is reasonably high.

$$z(t) = \frac{a y_{Av.stack}(t)}{\left(a - |y_{Av.stack}(t)|\right)} \qquad \text{(A2)}$$

This main lobe signal restoration works properly only for a boxcar window and omnidirectional elements. For real elements with innate directivities the initial scaling and subsequent restoration will be considerably less exact – probably a fundamental limitation on the effectiveness of this technique.

The overall idea is to give more weight to properly phase coherent higher amplitude signals in the beamforming process than if a strictly linear response beamformer were employed. It will be noted that, like the N^{th}- root stack, this technique behaves like a conventional analog noise limiter for high amplitude signals with a soft cutoff amplitude asymptotic to the value of a. Very high amplitude incoherent reverberation noise is strongly clipped pre-stack, then further attenuated in stacking, so that the stacked reverberation falls below the post-stack restoration threshold. However, the soft cutoff allows a degree of post-stack restoration of legitimate high amplitude stack-coherent signals. For low amplitude signals the effect is minimal and the signals are processed in essentially linear form.